RESISTANCE TO MOTION IN TURBULENT FLOW OF LIQUID FILM AND OF GAS IN VERTICAL TUBES

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Expressions are obtained for the pressure losses and the friction coefficient of the film by solving separate equations of motion for each phase. Theoretical and experimental results are compared.

An efficient method of improving heat- and mass-exchange processes is to transmit them in thin layers. A simultaneous flow of liquid film and of gas in a vertical tube can be described by a system of equations [1] which in abbreviated notation can be written as follows [2]:

$$\frac{\partial}{\partial r}(rv_{ir}) + \frac{\partial}{\partial x}(rv_{ix}) = 0; \tag{1}$$

$$\rho_i\left(v_{ir}\frac{\partial v_{ix}}{\partial r} + v_{ix}\frac{\partial v_{ix}}{\partial x}\right) = -\frac{\partial P}{\partial x} - \frac{1}{r}\cdot\frac{\partial}{\partial r}(r\tau_i) + [(2-i)\rho_1 - (-1)^{i-1}\rho_2]g. \tag{2}$$

By introducing the variable $y = R - r$ and transforming Eq. (2) with the aid of (1), one easily obtains

$$\rho_i\left\{(R-y)\frac{\partial v_{ix}^2}{\partial x} - \frac{\partial}{\partial y}[(R-y)v_{ir}v_{ix}]\right\} = -(R-y)\frac{\partial P}{\partial x} + \frac{\partial}{\partial y}[(R-y)\tau_i] + g[(2-i)\rho_1 - (-1)^{i-1}\rho_2](R-y). \tag{3}$$

Our solution of Eq. (3) must satisfy the following boundary conditions:

$$v_{1x} = v_{1r} = 0; \quad \tau_i = \tau_0 \ \text{for} \ y = 0;$$
$$v_{1x} = v_{2x} = v_\delta; \quad \tau_i = \tau_\delta \ \text{for} \ y = \delta. \tag{4}$$

From the relations (3) and (4) one can easily establish a relationship between the pressure gradient and the tangential stress on the flow boundaries. If for Eqs. (3) one refers to liquid and gas components, integrates them with respect to y from 0 to δ and from δ to R, respectively, and combines the results, one can find

$$\frac{\partial P}{\partial x} = -\frac{2\tau_0}{R} + \left[(\rho_1-\rho_2)g - \rho_1\frac{\partial \bar{v}_{1x}^2}{\partial x}\right]\frac{\delta(2R-\delta)}{R^2} + \rho_2\left(g - \frac{\partial \bar{v}_{2x}^2}{\partial x}\right)\frac{(R-\delta)^2}{R^2}; \tag{5}$$

$$\tau_\delta = 0.5(R-\delta)\left\{\frac{2\tau_0}{R} + \left[\rho_2\left(2g - \frac{\partial \bar{v}_{2x}^2}{\partial x}\right) + \rho_1\left(\frac{\partial \bar{v}_{1x}^2}{\partial x} - g\right)\right]\frac{\delta(2R-\delta)}{R^2}\right\}. \tag{6}$$

In the case of turbulent motion the tangential stresses are given by the relations

$$\tau_i = \rho_i\left(\nu_i + l_i^2\frac{dv_{ix}}{dy}\right)\frac{dv_{ix}}{dy}, \tag{7}$$

where the mixing path length l_i which appears in (7) is found by adopting a model of turbulent mixing of Van Driest [1, 3] modernized by Spalding.

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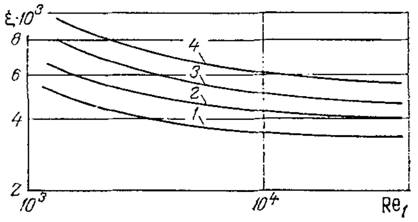

Fig. 1

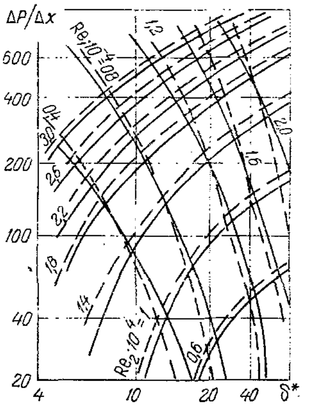

Fig. 2

Fig. 1. Resistance coefficients evaluated from the formula (12): 1) $Re_2 \cdot 10^{-4} = 1.4$; 2) 1.8; 3) 2.2; 4) 2.6.

Fig. 2. Comparison of pressure losses from the formulas (12) and (5) with those obtained experimentally; $\frac{\Delta P}{\Delta x}$, N/m^3.

For one dimensional steady flow, Eq. (3) applied to the liquid phase is brought to the form

$$\left(l_1^+ \frac{dv_{1x}^+}{dy^+}\right)^2 + \frac{4\delta^+}{Re_1}\cdot\frac{dv_{1x}^+}{dy^+} + \left[\frac{\tau_0}{\rho_1 \bar{v}_{1x}^2} + \frac{\rho_1-2\rho_2}{2\rho_1}\cdot\frac{(1-\delta^+)^2}{\delta^+ Fr_1}\right]\frac{y^+(2-y^+)}{1-y^+} - \frac{\tau_0}{\rho_1\bar{v}_{1x}^2}\cdot\frac{1}{1-y^+} = 0 \tag{8}$$

if the expressions (5) and (7) are used after transformations in dimensionless form.

In the latter expression one adopts the mean flow velocity in the film as a measure of velocity and the tube radius as a linear measure. Equation (8) is quadratic for the derivative $\frac{dv_{1x}^+}{dy^+}$ and can easily be solved for this derivative. However, it is still not known what sign (plus or minus) should be set in front of the square root. It is, therefore, expedient to introduce in (8) the change $u = \frac{dv_{1x}^+}{dy^+}$ and to differentiate it with respect to y^+; the latter yields the equation

$$\left(u + \frac{A}{2}\right)\frac{du}{dy^+} + \frac{u}{2}\cdot\frac{dA}{dy^+} + \frac{1}{2}\cdot\frac{dB}{dy^+} = 0, \tag{9}$$

with the notation

$$B = \left[\frac{\tau_0}{\rho_1\bar{v}_{1x}^2} + \frac{\rho_1-2\rho_2}{2\rho_1}\cdot\frac{(1-\delta^+)^2}{\delta^+ Fr_1}\right]\frac{y^+(2-y^+)}{(1-y^+)l_1^{+2}} - \frac{\tau_0}{\rho_1\bar{v}_{1x}^2}\cdot\frac{1}{(1-y^+)l_1^{+2}}; \quad A = \frac{4\delta^+}{Re_1 l_1^{+2}}.$$

The expression (9) is a particular case of the Abel equation of the second kind; after the restitution of the original variables its solution is

$$\frac{dv_{1x}^+}{dy^+} l_1^{+2} = -\frac{2\delta^+}{Re_1} + \left\{\frac{\tau_0}{\rho_1\bar{v}_{1x}^2}\cdot\frac{l_1^{+2}}{1-y^+} - \left[\frac{\tau_0}{\rho_1\bar{v}_{1x}^2} + \frac{(\rho_1-2\rho_2)(1-\delta^+)^2}{2\rho_1\delta^+ Fr_1}\right]\frac{y^+(2-y^+)l_1^{+2}}{1-y^+} + \frac{4\delta^{+2}}{Re_1^2}\right\}^{1/2}. \tag{10}$$

To find the derivative of the axial velocity of the gas flow component one must set i = 2 in Eq. (3) and carry out operations similarly as above; this results in the following:

$$\frac{dv_{2x}^+}{dy^+} l_2^{+2} = \frac{1-\delta^+}{Re_2} + \left\{\frac{(y^+-\delta^+)(2-\delta^+-y^+)l_2^{+2}}{2(1-y^+)}\left[-\frac{2\tau_0}{\rho_1\bar{v}_{1x}^2}\cdot\frac{\rho_1\bar{v}_{1x}^2}{\rho_2\bar{v}_{2x}^2} + \frac{\rho_1-2\rho_2}{2\rho_2}\cdot\frac{\delta^+(2-\delta^+)}{(1-\delta^+)Fr_2}\right] + \frac{[(1-\delta^+)l_2^+]^2}{2(1-y^+)}\left[-\frac{2\tau_0}{\rho_1\bar{v}_{1x}^2}\cdot\frac{\rho_1\bar{v}_{1x}^2}{\rho_2\bar{v}_{2x}^2} + \frac{\rho_1-\rho_2}{2\rho_2}\cdot\frac{\delta^+(2-\delta^+)}{(1-\delta^+)Fr_2} + \frac{1-\delta^+}{2Fr_2}\right] + \left(\frac{1-\delta^+}{Re_2}\right)^2\right\}^{1/2}. \tag{11}$$

In the above expression the mean gas velocity $\bar{v}_{2x}$ has been adopted as the velocity scale unit.

The condition that the tangential components (4) are continuous is valid on the phase separation boundary; the latter is used to determine from the formulas (10) and (11) the resistance coefficient

$$\xi = \frac{2\tau_0}{\rho_1 \bar{v}_{1x}^2} = \frac{1}{a_1}\left\{-2f - \frac{4a_2a_3}{a_1\mathrm{Re}_1^2} \pm \left[\left(f + \frac{4a_2a_3}{a_1\mathrm{Re}_1^2}\right)^2 - 4f^2 + \frac{16a_2}{\mathrm{Re}_1^2}\left(\frac{a_4}{\mathrm{Re}_2^2} + \frac{a_5}{4\mathrm{Fr}_2}\right)\right]^{1/2}\right\}, \tag{12}$$

where

$$f = \left[\frac{a_6}{\mathrm{Fr}_1} + \frac{a_7a_8}{\mathrm{Re}_1^2} - \left(\frac{a_4}{\mathrm{Re}_2^2} + \frac{a_5}{4\mathrm{Fr}_2}\right)\left(\frac{\mathrm{Re}_2}{\mathrm{Re}_1}\right)^2\right];$$

$$a_1 = a_3 + \frac{1-\delta^+}{l_{1\delta}^{+2}};\quad a_2 = \left(\frac{2\delta^+}{l_{1\delta}^{+2}} - a_7\right)^2;\quad a_3 = b\alpha_1;$$

$$a_4 = \alpha_1^2\alpha_2;\quad a_5 = \alpha_1\alpha_2\alpha_3;\quad a_6 = \frac{(\rho_1 - 2\rho_2)(2-\delta^+)(1-\delta^+)}{\rho_1 l_{1\delta}^{+2}};$$

$$a_7 = \frac{2b\delta^+}{l_{2\delta}^{+2}};\quad a_8 = \frac{4\delta^+}{l_{1\delta}^{+2}} - a_7;\quad \alpha_1 = \frac{1-\delta^+}{l_{2\delta}^{+2}};\quad \alpha_2 = \frac{2b\delta^+}{1-\delta^+};$$

$$\alpha_3 = \frac{\delta^+(\rho_1-\rho_2)(2-\delta^+)}{\rho_2(1-\delta^+)} + 1 - \delta^+;\quad b = \frac{\rho_2}{\rho_1}\left(\frac{\nu_2}{\nu_1}\right)^2.$$

In Fig. 1 computation results are shown of the friction coefficients in the case of a sinking annular flow versus Reynolds numbers for both phases. It can be seen from Fig. 1 that a decisive effect on the values of the resistance coefficients is exerted by the gas component. It has up until now been very difficult to compare the theoretically obtained results shown in Fig. 1 with the experimental data, since the direct measurements of tangential stresses in a two-phase flow are very complex. In our case the simplest comparison, as well as the most convenient one, is between the theoretical and experimental data on head-pressure losses. In Fig. 2 specific pressure losses are shown versus the dimensionless film thickness δ^* and the Reynolds numbers for both phases obtained with the aid of (5) and (12) (continuous lines). Experimental values are also shown measured on a setup [4] for an annular air–water sinking flow in a tube of 30 mm diameter (dashed lines). It can be seen from the diagram that there is a satisfactory agreement between the theoretical and experimental data on pressure losses. For given flow rates the relations shown in Fig. 2 enable one to find not only the pressure gradient, but also the corresponding thickness of the liquid film. It should be mentioned here that the theoretical and experimental results were obtained under the conditions of purely film state of the flow when the entire liquid moves in the film.

NOTATION

r, x, radial and axial cylindrical coordinates; R, tube radius; δ, film thickness; l_i, mixing path length; v_{ir}, v_{ix}, corresponding velocity projections; ρ_i, density; ν_i, kinematic viscosity; P, pressure; τ_i, frictional stress; g, gravity acceleration; $\mathrm{Re}_1 = 4\bar{v}_{1x}\delta\nu_1^{-1}$; $\mathrm{Re}_2 = 2\bar{v}_{2x}(R-\delta)\nu_2^{-1}$; $\mathrm{Fr}_1 = \bar{v}_{1x}^2(g\delta)^{-1}$; $\mathrm{Fr}_2 = \bar{v}_{2x}^2[2g(R-\delta)]^{-1}$; $\delta^* = \delta g^{1/3}\nu^{-2/3}$. Indices: i = 1, liquid components; i = 2, gas components; δ, 0, values on the film surface and on the tube wall; the bar on top shows the mean values, and the symbol «+», dimensionless quantities of the type $v_{ix}^+ = v_{ix}/\bar{v}_{ix}$; $\delta^+ = \delta/R$ and others.

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